Atomic Masses and Fundamental Constants 5

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A Laboratory Experiment to Measure the Time Variation of Newton's Gravitational Constant

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1. Introduction

The possibility that Newton's gravitational constant, G, varies in time was suggested theoretically by Dirac on numerological grounds.[1] Brans and Dicke[2] have proposed a specific scalar-tensor theory which predicts a variation, with time, in G; and it is also true that most metric theories of gravity[3] make such a prediction. As evaluated thus far the most serious of these theories predict that $\dot{G}/G \sim 10^{-10}$ to 10^{-11} per year.

Because of the small value predicted for $\dot{G}/G$ experiment has not had any effective bearing on this problem until recently. In fact, no direct laboratory experiment has reported on this, and only a few indirect results from astronomical observations are available. In particular, the most recent implications from analysis of radar ranging to inner planets[4] has set $\dot{G}/G = (4 \pm 8) \times 10^{-11}$ per year. Lunar occultation experiments[5] have given a value $(-7.5 \pm 2.7) \times 10^{-11}$ per year. Another result,[6] the rate of slowdown of the period of pulsar JP 1953, infers an upper limit of 7×10^{-10} per year for $\dot{G}/G$. The improvement, in time, of the radar-ranging precision is automatic and, thus, offers hope of a serious test at the level of 10^{-11} in a few years.

It is true, regardless of the astronomical results, that a laboratory test of $\dot{G}/G$ at or near the level of 10^{-11} per year would be scientifically of high interest. Such an experiment is what we will describe.

A fedback Cavendish balance configuration, optimized for sensitivity and stability rather than absolute accuracy, can, in principle, provide the requisite precision. It would be critically damped, cooled to $\lesssim 1°$ K and would rotate at a constant angular velocity of a few rpm. The previous experiment which most nearly approaches this one, in terms of properties and problems, is the Eotvos-type of experiment by Dicke.[7)]

2. General Features

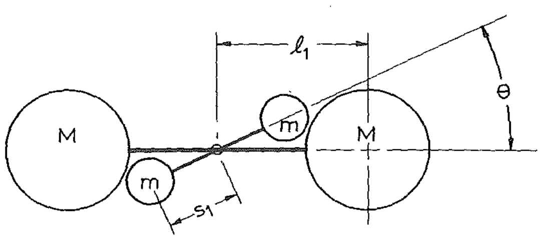
Large and small dumbells (Fig. 1) provide a gravitational mass quadrupole-quadrupole interaction. The torsional restraint for such a system will most likely not be a torsion fiber, thus avoiding questions of solid state stability of the fiber. It would be desirable, in fact, to use centrifugal forces for the restraining torque, but we have not found satisfactory configuration for that. Present plans are to use a magnetic support and an eddy current drag for the main part of the restraint. This, admittedly, puts electromagnetic forces into the question of any observed variation, but it seems our best present option. Calculations indicate that special high-resistance alloys can be made which will have resistence with the needed stability and temperature independence.

Scaling of the experiment is important in a number of the aspects of design. The gravitational torque experienced by the small mass system, in the rotating coordinate frame, is factored into a scaling factor f and an angle function $F(x,\theta)$:

$$T_g = f\,F(x,\theta); \quad f = 2Gm\,m\,\frac{s_1^2}{\ell_1^3} = \frac{1}{8}\frac{GQq}{\ell_1^5},$$

$$F(x,\theta) = \frac{\sin\theta}{x}\left\{\frac{1}{(1+x^2-2x\cos\theta)^{3/2}} - \frac{1}{(1+x^2+2x\cos\theta)^{3/2}}\right\} \quad (1)$$

where Q = the quadrupole moment of the large mass, q = the quadrupole



$T_g = f\,F(x,\Theta)$ $\quad x = \frac{s_1}{\ell_1}$

$f = 2GMm\,\frac{s_1^2}{\ell_1^3}$

$F(x,\Theta) = \frac{\sin\Theta}{x}\left\{\frac{1}{(1+x^2-2x\cos\Theta)^{3/2}} - \frac{1}{(1+x^2+2x\cos\Theta)^{3/2}}\right\}$

Fig. 1. Torsion balance configuration showing the factorization of the torque into a scaling part f and an angle-dependent part $F(x,\theta)$

moment of the small mass, and other symbols are shown in Fig. 1. If ℓ_1 = 10 cm, m = 0.5 Kg, M = 5 Kg, and x = 0.5, then f $\sim$ 0.01 dyne-cm. Tungsten balls with these masses will fit into these dimensions, but very much less dense materials will not.

Our primary use of scaling is in connection with the signal-to-noise ratio. The signal due to a change in G is given by

$$d\theta = \frac{\delta T_g}{C} \tag{2}$$

where $\delta = \frac{dT_g}{T_g}$ = the fractional change in gravitational torque during the measuring period and C is the restoring torque coefficient.

For an angle-independent restraining torque, such as an eddy current drag or a fiber with many turns, the restoring torque coefficient C is purely gravitational, as can be seen from integrating equation (1) with respect to angle (Fig. 2). $V(x,\theta)$ provides the

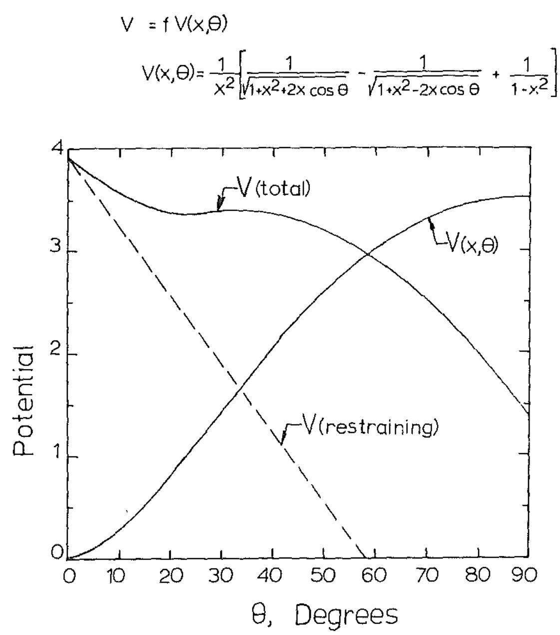


Fig. 2. Potential as a function of angle showing the gravitational and restraining parts.

curvature in the total potential function of angle and hence the restoration to an equilibrium angle. Thus, $C = \frac{\partial^2 V}{\partial \theta^2} = \frac{\partial T_g}{\partial \theta}$. This can be determined by a combination of the scaling factor f and the setting of the equilibrium angle θ_o by adjustment of the slope of the drag potential V(restraint). Once a factor f is chosen, varying θ_o changes C radically with small change in the equilibrium T_g.

The rms value of the thermal noise is given by the usual relation

$$\Delta\theta_n = \sqrt{\frac{kT}{C}}, \tag{3}$$

where k is Boltzmann's constant and T is the temperature.

The signal-to-noise ratio is therefore given by

$$S/N = \frac{d\theta}{\Delta\theta_n} W = \frac{\delta T_g}{\sqrt{CkT}} W = \sqrt{\frac{\delta T_g \, d\theta}{kT}} W = \frac{\delta T_g \, \tau}{2\pi \sqrt{IkT}} W \tag{4}$$

where W is a signal-averaging factor related to the way the final data is treated, τ is the natural period of the balance and I is the moment of inertia of the small mass system.

3. Design of a Cavendish Balance Experiment

Since W is independent of f, Eq. (4) shows that $S/N \sim \sqrt{f}$. That is, a larger experiment should improve the signal-to-noise ratio. Other factors such as the effect of external masses will favor a smaller experiment.

To use Eq. (4) to evaluate parameters of the experiment, we need to determine the signal-averaging factor W. An expression

$$W = \sqrt{\frac{S}{\tau_e}}, \tag{5}$$

is derived from elementary statistical considerations, assuming independent samples of the signal at intervals τ_e, the effective response time of the experiment. In the equation S is the total observation period.

This expression considers the fact that the rms value of fluctuations cannot be reduced below Eq. (3) by varying the damping or the moment of intertia, or by feedback. Feedback can, however, reduce the effective period τ_e and thereby increase the number of samples in time S.

If a reasonable experimental strategy is to observe changes in G of 10^{-11} per year in a two month period to an accuracy (S/N) of 10, this puts a great burden of precision on the experiment. Operating at 10 mK°, using positional and derivative feedback to greatly reduce the value of τ_e, and using masses and lengths as previously given, we arrive at the following parameters: $\tau = 2.44 \times 10^6$ sec (28 days), $C = 1.7 \times 10^{-7}$ dyne-cm/radian and $d\theta = 3.9 \times 10^{-7}$ radians. The accomplishment of this requires that certain feedback criteria and angle sensing sensitivity be met.

Predicted disturbances or drifts in the system are listed in Table I.

Table I: Sources of Disturbances or Drifts

A.	Internal
	1. Mass Stability
	2. Dimensional Stability
	3. Angle-sensor Stability
	4. Thermal Noise
	5. Electrostatic Variations
	6. Thermal Variations
	7. Pressure Variations
	8. Stability of Angular Velocity
B.	External
	1. Magnetic Field Variations
	2. Surrounding Mass Vibrations
	3. Ground Vibrations

Mass, dimensional, and angle-sensing stabilities to better than 10^{-12} per year are within present technology, since the system is cooled to $\lesssim 1°$ K. This cooling also establishes an adequate level of thermal noise and is expected to control thermal and electrostatic variations. At the present time, little is known about the variation of small electrostatic forces in a supercooled system, but superconductivity is obviously an important feature in this respect. A superconducting magnetic shield, essential in this experiment, will be a concommittant aspect of the cooled design.

Changes in gravitational gradients, such as tidal effects or extraneous masses, disturb the equilibrium point of the balance. In analogy with the torque calculation of Eq. (1) we can determine the anomalous torque T_a of some extraneous mass $\mathcal{M}$ at a distance r. Its relative magnitude is

$$\frac{T_a}{T_g} = \frac{\mathcal{M}}{2M} \cdot \frac{\ell_1^{\,3}}{r^3}. \qquad (6)$$

The instantaneous value of this is 10^{-12} for a 100 Kg man at 2100 meters with a system scaled as above. For this reason Dicke[7] used an octopole configuration to gain a higher inverse power of dependence on distance. In our experiment, however, rotation of the entire system with a period much shorter than the natural period of the balance provides an averaging and filtering function which is the equivalent of an off-resonance driving torque. Thus, the actual amplitude is reduced by a factor of approximately $\frac{\omega_o^{\,2}}{\omega^2 Q}$, where ω_o is the resonant frequency of the balance and ω is twice the rotation frequency. The expected parameters are such that $\omega \gtrsim 10^5\ \omega_o$ and Q should be $\sim$ unity (critical damping).

Vibrations are expected to be one of the most serious of the disturbances. The nature of the problem can be seen in Fig. 3, which is an electrical circuit analog. Due to the high sensitivity (small value of C) needed in the balance, the coupling between mass systems is extremely weak, represented by a high impedance Z_2. Isolation from ground (the driving point) and the large mass system will be as good as possible, but cannot be expected to lead to $Z_1 >> Z_2$. Methods used in gravitational wave detection will be employed to some extent, but the fact that our signal is nearly dc as compared with $\sim 10^3$ Hz adds complications. A feedback-corrected vibrational, isolating support table with response down to 0 Hz will be used and can be expected to give improvement of a factor of 30. Fortunately, the same filtering effect of rotation which applied to external mass effects will also apply to vibration. Judicious design is needed to assure the appropriate directional properties for the vibrational isolation.

Feedback in the balance will be used to help in many of these problems, for example, in the reduction of the effect of vibrations as in Fig. 3. If the impedances Z_1 and Z_2 are equal, the improvement is $\sim 1/2\ g_1 g_2$.

The system differential equation is

$$I\ddot{\theta} + K\dot{\theta} + C\theta = F(t), \qquad (7)$$

where the driving function F(t) includes all disturbances, as well as the signal. Feedback, in addition, is incorporated in F(t). Position-sensitive feedback, in effect, adds to the stiffness C, while derivative feedback can increase the damping to optimum without increasing noise.[8] In order to perform this with appropriately long time constants, a digital computer will be needed.

Angle sensing must be sensitive and stable, and must not contribute appreciable noise. Two methods are still being considered: laser interferometry and position-sensing based on the SQUID magnetometer. Either are technically capable of meeting the requirements, but the laser interferometry is perhaps simpler.

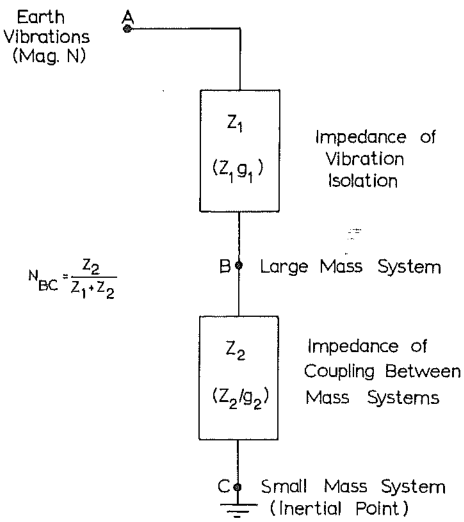


Fig. 3. Electrical circuit equivalent to the main vibration properties of the experiment.

A concept of design envisaged now is shown in Fig. 4. Four small mass systems in a symmetrical arrangement can provide intrinsic cancellation of vibrational effects, averaging of noise and internal testing of drift. The signals 1-4 can be summed electronically, mechanically (in part) or optically. Electronic summation of the four separate sets of data offers the best internal testing of drift but is the most complex.

Caution must be applied to any attemt to measure to a part in 10^{11}, but the intrinsic aspects of this design do not imply a limitation to our ability to achieve this.

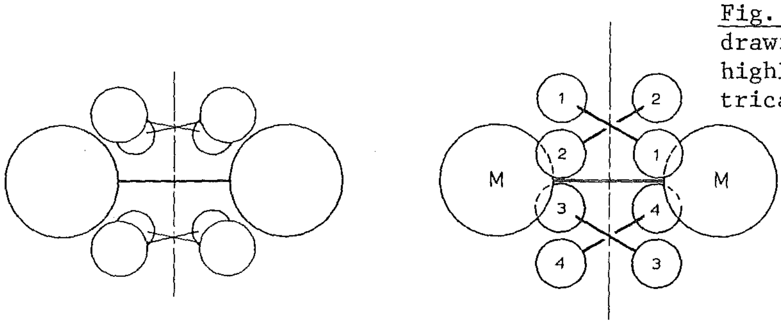


Fig. 4 Two drawings of a highly symmetrical system.

References

1. DIRAC, P.A.M., Nature 139, 323 (1937); Proc. Roy. Soc. A165, 199 (1938).

2. BRANS, C. and DICKE, R.H., Phys. Rev. 124, 925 (1961).

3. MISNER, CHARLES W., THORNE, KIP S., and WHEELER, JOHN ARCHIBALD, Gravitation (W. H. Freeman and Company, San Francisco, 1973), p. 1122.

4. COUNSELMAN, C.C. and SHAPIRO, IRWIN I., Private Communication

5. VAN FLANDERN, T.C., B.A.P.S. II, 20, 543 (1975).

6. COUNSELMAN, C.C. and SHAPIRO, I.I., Science 162, 352 (1968); RICHARDS, D.W., RANKIN, J.M., and ZESSIG, G.A., Nature 251, 37 (1974).

7. ROLL, P.G., KROTKOV, R. and DICKE, R.H., Annals of Physics 26, 442 (1964).

8. MCCOMBIE, C.W., Repts. Prog. in Phys. 16, 266 (1953).

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